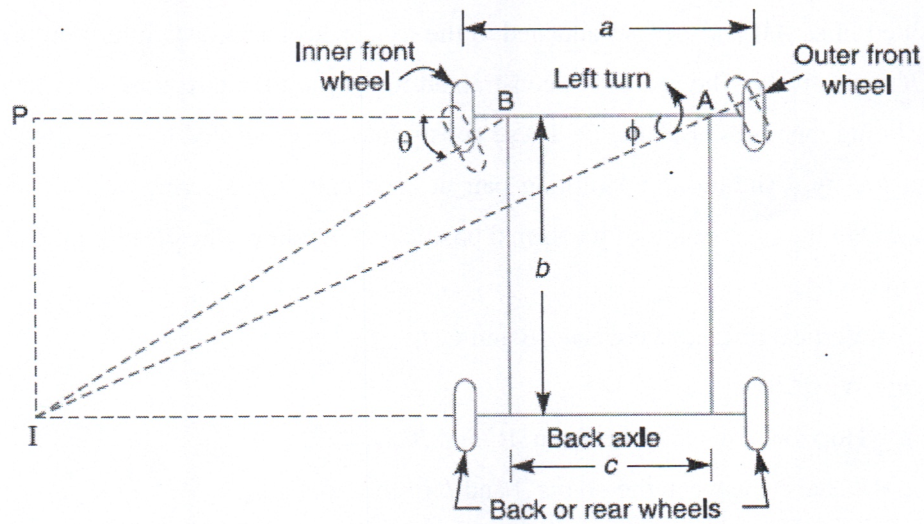
UNIT-II

Day-7

Derive the equation for correct steering.

The steering gear mechanism is used for changing the direction of two or more of the wheel axles with reference to the chassis, so as to move the automobile in any desired path. Usually the two back wheels have a common axis, which is fixed in direction with reference to the chassis and the steering is done by means of the front wheels. In automobiles, the front wheels are placed over the front axles, which are pivoted at the points *A* and *B*, as shown in Fig. These points are fixed to the chassis. The back wheels are placed over the back axle, at the two ends of the differential tube. When the vehicle takes a turn, the front wheels along with the respective axles turn about the respective pivoted points. The back wheels remain straight and do not turn. Therefore, the steering is done by means of front wheels only.



In order to avoid skidding (*i.e.* slipping of the wheels sideways), the two front wheels must turn about the same instantaneous centre *I* which lies on the axis of the back wheels. If the instantaneous centre of the two front wheels do not coincide with the instantaneous centre of the back wheels, the skidding on the front or back wheels will definitely take place, which will cause more wear and tear of the tyres. Thus, the condition for correct steering is that all the four wheels must turn about the same instantaneous centre. The axis of the inner wheel makes a larger turning angle θ than the angle φ subtended by the axis of outer wheel.

Let a = Wheel track,

b = Wheel base, and

c = Distance between the pivots A and B of the front axle.

Now from triangle IBP,

$$\cot\theta = \frac{BP}{IP}$$

and from triangle IAP,

$$\cot\phi = \frac{AP}{IP} = \frac{AB + BP}{IP} = \frac{AB}{IP} + \frac{BP}{IP} = \frac{c}{b} + \cot\theta \qquad \ldots(\because IP = b)$$

$$\therefore \cot\phi - \cot\theta = c / b$$

This is the fundamental equation for correct steering. If this condition is satisfied, there will be no skidding of the wheels, when the vehicle takes a turn.

Explain Davis Steering Gear mechanism with neat sketch.

The Davis steering gear is shown in Fig. It is an exact steering gear mechanism. The slotted links *AM* and *BH* are attached to the front wheel axle, which turn on pivots *A* and *B* respectively. The rod *CD* is constrained to move in the direction of its length, by the sliding members at *P* and *Q*. These constraints are connected to the slotted link *AM* and *BH* by a sliding and a turning pair at each end. The steering is affected by moving *CD* to the right or left of its normal position. *C 'D'* shows the position of *CD* for turning to the left.

Let a = Vertical distance between AB and CD,

b = Wheel base,

d = Horizontal distance between AC and BD,

c = Distance between the pivots A and B of the front axle.

x = Distance moved by AC to $AC' = CC' = DD'$, and

α = Angle of inclination of the links AC and BD, to the vertical.

From triangle $A\,A'\,C'$,

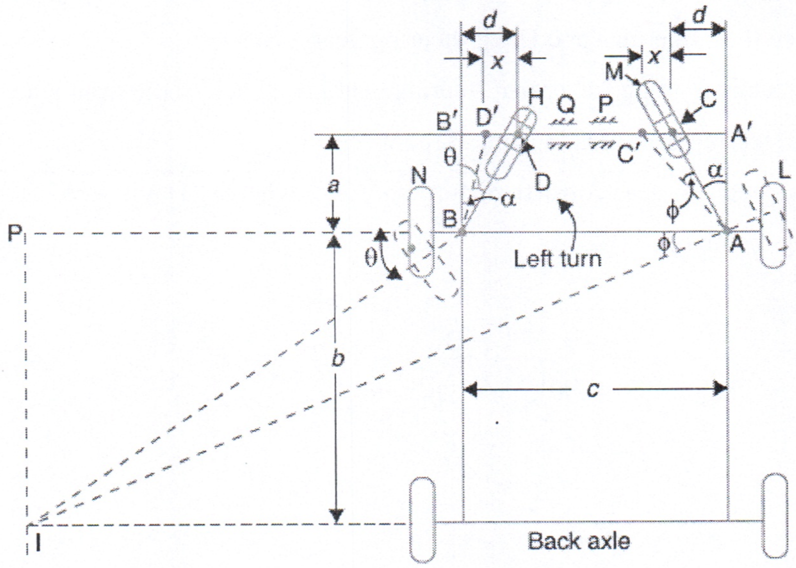
$$\tan(\alpha + \phi) = \frac{A'C'}{A\,A'} = \frac{d + x}{a}$$

From triangle $AA'C$,

$$\tan\alpha = \frac{A'C}{AA'} = \frac{d}{a}$$

From triangle $BB'D'$,

$$\tan(\alpha - \theta) = \frac{B'D'}{BB'} = \frac{d - x}{a}$$



We know that $\tan(\alpha + \phi) = \dfrac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}$

or $$\frac{d + x}{a} = \frac{d/a + \tan\phi}{1 - d/a \times \tan\phi} = \frac{d + a\tan\phi}{a - d\tan\phi}$$

...[From equations (*i*) and (*ii*)]

$$(d + x)(a - d\tan\phi) = a(d + a\tan\phi)$$

$$a.d - d^2\tan\phi + a.x - d.x\tan\phi = a.d + a^2\tan\phi$$

$$\tan\phi(a^2 + d^2 + d.x) = ax \quad \text{or} \quad \tan\phi = \frac{a.x}{a^2 + d^2 + d\,x} \qquad \text{...}(iv)$$

Similarly, from $\tan(\alpha - \theta) = \dfrac{d - x}{a}$, we get

$$\tan\theta = \frac{ax}{a^2 + d^2 - d\,x} \qquad \text{...}(v)$$

We know that for correct steering,

$$\cot\phi - \cot\theta = \frac{c}{b} \quad \text{or} \quad \frac{1}{\tan\phi} - \frac{1}{\tan\theta} = \frac{c}{b}$$

$$\frac{a^2 + d^2 + d.x}{a.x} - \frac{a^2 + d^2 - d.x}{a.x} = \frac{c}{b}$$

...[From equations (*iv*) and (*v*)]

or $$\frac{2d.x}{a.x} = \frac{c}{b} \quad \text{or} \quad \frac{2d}{a} = \frac{c}{b}$$

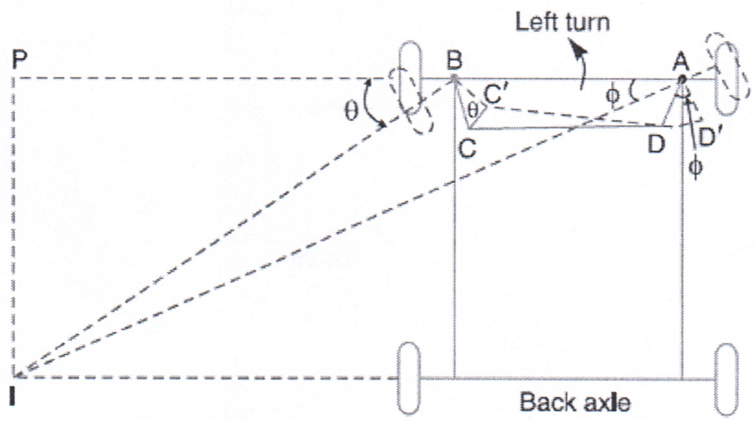
$$\therefore \quad 2\tan\alpha = \frac{c}{b} \quad \text{or} \quad \tan\alpha = \frac{c}{2b} \qquad ...(\because d/a = \tan\alpha)$$

Explain Ackerman Steering Gear mechanism.

The Ackerman steering gear mechanism is much simpler than Davis gear. The difference between the Ackerman and Davis steering gears are :

1. The whole mechanism of the Ackerman steering gear is on back of the front wheels; whereas in Davis steering gear, it is in front of the wheels.

2. The Ackerman steering gear consists of turning pairs, whereas Davis steering gear consists of sliding members.



In Ackerman steering gear, the mechanism *ABCD* is a four bar crank chain, as shown in Fig.

The shorter links *BC* and *AD* are of equal length and are connected by hinge joints with front wheel axles. The longer links *AB* and *CD* are of unequal length. The following are the only three positions for correct steering.

1. When the vehicle moves along a straight path, the longer links *AB* and *CD* are parallel and the shorter links *BC* and *AD* are equally inclined to the longitudinal axis of the vehicle, as shown by firm lines in Fig.

2. When the vehicle is steering to the left, the position of the gear is shown by dotted lines in Fig. In this position, the lines of the front wheel axle intersect on the back wheel axle at *I*, for correct steering.

3. When the vehicle is steering to the right, the similar position may be obtained.

In order to satisfy the fundamental equation for correct steering, the links *AD* and *DC* are suitably proportioned. The value of θ and φ may be obtained either graphically or by calculations.

Previous questions

1) Derive the condition of correct steering in the case of an automobile. (2013-April-Set-1)

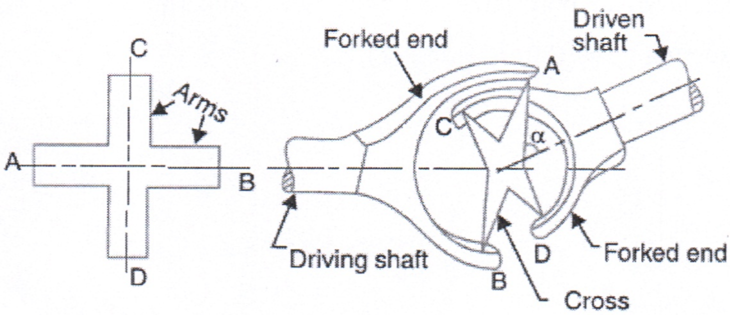
2) Draw a neat sketch of Davis steering gear and show that the condition of correct steering (2013-April-Set-4)

3) Draw a neat sketch of Ackerman steering gear and show that the condition of correct steering is satisfied for only three positions. Why Ackerman steering gear preferred over Davis steering gear? (2014-Jan-Set-1)

Day-8

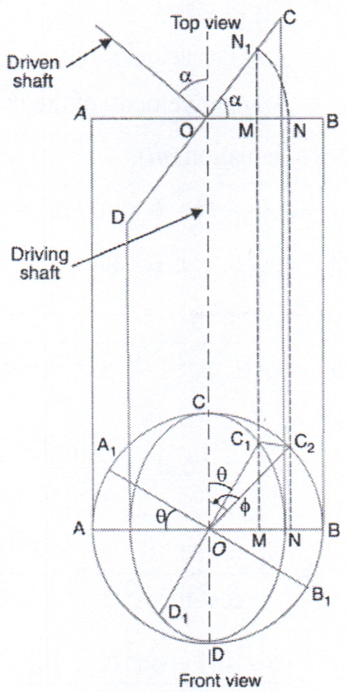
Explain Hooke's Joint:

A Hooke's joint is used to connect two shafts, which are intersecting at a small angle, as shown in Fig. The end of each shaft is forked to U-type and each fork provides two bearings for the arms of a cross. The arms of the cross are perpendicular to each other. The motion is transmitted from the driving shaft to driven shaft through a cross. The inclination of the two shafts may be constant, but in actual practice it varies, when the motion is transmitted. The main application of the Universal or Hooke's joint is found in the transmission from the gear box to the differential or back axle of the automobiles. It is also used for transmission of power to different spindles of multiple drilling machine. It is also used as a knee joint in milling machines.



Derive the Ratio of the Shafts Velocities

The top and front views connecting the two shafts by a universal joint are shown in Fig. Let the initial position of the cross be such that both arms lie in the plane of the paper in front view, while the arm *AB* attached to the driving shaft lies in the plane containing the axes of the two shafts. Let the driving shaft rotates through an angle θ, so that the arm *AB* moves in a circle to a new position A_1 B_1 as shown in front view. The arm *CD* will also move in a circle of the same size. This circle when projected in the plane of paper appears to be an ellipse. Therefore the arm *CD* takes new position C_1D_1 on the ellipse, at an angle θ. But the true angle must be on the circular path. To find the true angle, project the point C_1 horizontally to intersect the circle at C_2. Therefore the angle COC_2 (equal to φ) is the true angle turned by the driven shaft. Thus when the driving shaft turns through an angle θ, the driven shaft turns through an angle φ. It may be noted that it is not necessary that φ may be greater than θ or less than θ. At a particular point, it may be equal to θ.



In triangle OC_1M, $\angle OC_1M = \theta$

$$\therefore \qquad \tan\theta = \frac{OM}{MC_1} \qquad ...(i)$$

and in triangle OC_2N, $\angle OC_2N = \phi$

$$\therefore \qquad \tan\phi = \frac{ON}{NC_2} = \frac{ON}{MC_1}$$

Dividing equation (*i*) by (*ii*),

$$\frac{\tan\theta}{\tan\phi} = \frac{OM}{MC_1} \times \frac{MC_1}{ON} = \frac{OM}{ON}$$

But $\qquad OM = ON_1 \cos\alpha = ON \cos\alpha$

...(where α = Angle of inclination of the driving and driven shafts)

$$\therefore \quad \frac{\tan\theta}{\tan\phi} = \frac{ON\cos\alpha}{ON} = \cos\alpha$$

or $\quad \tan\theta = \tan\phi \cdot \cos\alpha$ ...(*iii*)

Let $\quad \omega$ = Angular velocity of the driving shaft = $d\theta / dt$

ω_1 = Angular velocity of the driven shaft = $d\phi / dt$

Differentiating both sides of equation (*iii*),

$$\sec^2\theta \times d\theta / dt = \cos\alpha \cdot \sec^2\phi \times d\phi / dt$$

$$\sec^2\theta \times \omega = \cos\alpha \cdot \sec^2\phi \times \omega_1$$

$$\therefore \quad \frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\cos\alpha.\sec^2\phi} = \frac{1}{\cos^2\theta.\cos\alpha.\sec^2\phi} \quad \text{...(iv)}$$

We know that $\quad \sec^2\phi = 1 + \tan^2\phi = 1 + \dfrac{\tan^2\theta}{\cos^2\alpha}$ ...[From equation (*iii*)]

$$= 1 + \frac{\sin^2\theta}{\cos^2\theta.\cos^2\alpha} = \frac{\cos^2\theta.\cos^2\alpha + \sin^2\theta}{\cos^2\theta.\cos^2\alpha}$$

$$= \frac{\cos^2\theta(1-\sin^2\alpha) + \sin^2\theta}{\cos^2\theta.\cos^2\alpha} = \frac{\cos^2\theta - \cos^2\theta.\sin^2\alpha + \sin^2\theta}{\cos^2\theta.\cos^2\alpha}$$

$$= \frac{1-\cos^2\theta.\sin^2\alpha}{\cos^2\theta.\cos^2\alpha} \quad \text{...}(\because \cos^2\theta + \sin^2\theta = 1)$$

Substituting this value of $\sec^2\phi$ in equation (*iv*), we have veloity ratio,

$$\frac{\omega_1}{\omega} = \frac{1}{\cos^2\theta.\cos\alpha} \times \frac{\cos^2\theta.\cos^2\alpha}{1-\cos^2\theta.\sin^2\alpha} = \frac{\cos\alpha}{1-\cos^2\theta.\sin^2\alpha} \quad \text{...(v)}$$

Note: If $\quad N$ = Speed of the driving shaft in r.p.m., and

N_1 = Speed of the driven shaft in r.p.m.

Then the equation (*v*) may also be written as

$$\frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2\theta.\sin^2\alpha}.$$

Determine Maximum and Minimum Speeds of Driven Shaft

$$\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1 - \cos^2\theta.\sin^2\alpha} \quad \text{or} \quad \omega_1 = \frac{\omega.\cos\alpha}{1 - \cos^2\theta.\sin^2\alpha} \quad ...(i)$$

The value of ω_1 will be maximum for a given value of α, if the denominator of equation (*i*) is minimum. This will happen, when

$$\cos^2\theta = 1, \quad i.e. \text{ when } \theta = 0°, 180°, 360° \text{ etc.}$$

∴ Maximum speed of the driven shaft,

$$\omega_{1(max)} = \frac{\omega\cos\alpha}{1 - \sin^2\alpha} = \frac{\omega\cos\alpha}{\cos^2\alpha} = \frac{\omega}{\cos\alpha} \quad ...(ii)$$

or $$N_{1(max)} = \frac{N}{\cos\alpha} \quad \text{...(where } N \text{ and } N_1 \text{ are in r.p.m.)}$$

Similarly, the value of ω_1 is minimum, if the denominator of equation (*i*) is maximum. This will happen, when $(\cos^2\theta \,.\, \sin^2\alpha)$ is maximum, or

$$\cos^2\theta = 0, \; i.e. \text{ when } \theta = 90°, 270° \text{ etc.}$$

∴ Minimum speed of the driven shaft,

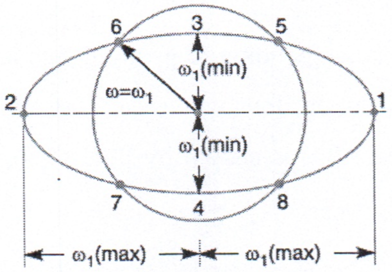
$$\omega_{1\,(min)} = \omega\cos\alpha$$

or $$N_{1\,(min)} = N\cos\alpha \quad \text{...(where } N \text{ and } N_1 \text{ are in r.p.m.)}$$

Explain about Polar Diagram with neat sketch showing silent points

Fig, shows the polar diagram depicting the salient features of the driven shaft speed.

From above, we see that



1. For one complete revolution of the driven shaft, there are two points *i.e.* at 0° and 180° as shown by points 1 and 2 in Fig. where the speed of the driven shaft is maximum and there are two points *i.e.* at 90° and 270° as shown by point 3 and 4 where the speed of the driven shaft is minimum.

2. Since there are two maximum and two minimum speeds of the driven shaft, therefore there are four points when the speeds of the driven and driver shaft are same. This is shown by points, 5,6,7 and 8 in Fig.

3. Since the angular velocity of the driving shaft is usually constant, therefore it is represented by a circle of radius ω. The driven shaft has a variation in angular velocity, the maximum value being $\omega/\cos\alpha$ and minimum value is $\omega\cos\alpha$. Thus it is represented by an ellipse of semi-major axis $\omega/\cos\alpha$ and semi-minor axis $\omega\cos\alpha$, as shown in Fig

Condition for Equal Speeds of the Driving and Driven Shafts

We have already discussed that the ratio of the speeds of the driven and driving shafts is

$$\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1-\cos^2\theta.\sin^2\alpha} \quad \text{or} \quad \omega = \frac{\omega_1(1-\cos^2\theta.\sin^2\alpha)}{\cos\alpha}$$

For equal speeds, $\omega = \omega_1$, therefore

$$\cos\alpha = 1-\cos^2\theta\,.\,\sin^2\alpha \quad \text{or} \quad \cos^2\theta\,.\,\sin^2\alpha = 1-\cos\alpha$$

$$\cos^2\theta = \frac{1-\cos\alpha}{\sin^2\alpha} \qquad \ldots(i)$$

We know that $\sin^2\theta = 1-\cos^2\theta = 1-\frac{1-\cos\alpha}{\sin^2\alpha} = 1-\frac{1-\cos\alpha}{1-\cos^2\alpha}$

$$= 1-\frac{1-\cos\alpha}{(1+\cos\alpha)(1-\cos\alpha)} = 1-\frac{1}{1+\cos\alpha} = \frac{\cos\alpha}{1+\cos\alpha} \qquad \ldots(ii)$$

Dividing equation (*ii*) by equation (*i*),

$$\frac{\sin^2\theta}{\cos^2\theta} = \frac{\cos\alpha}{1+\cos\alpha} \times \frac{\sin^2\alpha}{1-\cos\alpha}$$

$$\tan^2\theta = \frac{\cos\alpha\sin^2\alpha}{1-\cos^2\alpha} = \frac{\cos\alpha.\sin^2\alpha}{\sin^2\alpha} = \cos\alpha$$

$$\therefore \quad \tan\theta = \pm\sqrt{\cos\alpha}$$

There are two values of θ corresponding to positive sign and two values corresponding to negative sign. Hence, there are four values of θ, at which the speeds of the driving and driven shafts are same. This is shown by point 5, 6, 7 and 8 in Fig above polar diagram.

Determine Angular Acceleration of the Driven Shaft

We know that $$\omega_1 = \frac{\omega\cos\alpha}{1-\cos^2\theta.\sin^2\alpha} = \omega.\cos\alpha(1-\cos^2\theta.\sin^2\alpha)^{-1}$$

Differentiating the above expression, we have the angular acceleration of the driven shaft,

$$\frac{d\,\omega_1}{dt} = \omega\cos\alpha\left[-1(1-\cos^2\theta\sin^2\alpha)^{-2}\times(2\cos\theta\sin\theta\sin^2\alpha)\right]\frac{d\,\theta}{dt}$$

$$= \frac{-\omega^2\cos\alpha\times\sin 2\theta.\sin^2\alpha}{(1-\cos^2\theta\sin^2\alpha)^2} \qquad ...(i)$$

...($2\cos\theta\sin\theta = \sin 2\theta$, and $d\theta/dt = \omega$)

The negative sign does not show that there is always retardation. The angular acceleration may be positive or negative depending upon the value of $\sin 2\theta$. It means that during one complete revolution of the driven shaft, there is an angular acceleration corresponding to increase in speed of ω_1 and retardation due to decrease in speed of ω_1. For angular acceleration to be maximum, differentiate $d\omega_1 / dt$ with respect to θ and equate to zero. The result is approximated as

$$\cos 2\theta = \frac{\sin^2\alpha(2-\cos^2 2\theta)}{2-\sin^2\alpha}$$

Note: **If the value of α is less than 30°, then $\cos 2\theta$ may approximately be written as**

$$\cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha}$$

Previous questions

1) What is a Hooke's Joint? Show that for a Hooke's Joint $\tan\theta = \tan\varphi\cos\alpha$

2) Derive an expression for the speed ratio of driven shaft to the driving shaft of the Hooke's joint. (2013-April-Set-2)

3) Sketch the polar diagram of a Hooke's Joint and mark its salient points. (2013-April-Set-3)

4) Explain why two Hooke's joints are used to transmit motion from the engine to the differential of an automobile? (2013-April-Set-4)

Day-9

1) Two shafts with an included angle of 160° are connected by a Hooke's joint. The driving shaft runs at a uniform speed of 1500 r.p.m. The driven shaft carries a flywheel of mass 12 kg and 100 mm radius of gyration. Find the maximum angular acceleration of the driven shaft and the maximum torque required.

Solution. Given : $\alpha = 180° - 160° = 20°$; $N = 1500$ r.p.m.; $m = 12$ kg ; $k = 100$ mm $= 0.1$ m

We know that angular speed of the driving shaft,

$$\omega = 2\pi \times 1500 / 60 = 157 \text{ rad/s}$$

and mass moment of inertia of the driven shaft,

$$I = m.k^2 = 12\,(0.1)^2 = 0.12 \text{ kg - m}^2$$

Maximum angular acceleration of the driven shaft

Let $d\omega_1 / dt$ = Maximum angular acceleration of the driven shaft, and

θ = Angle through which the driving shaft turns.

We know that, for maximum angular acceleration of the driven shaft,

$$\cos 2\theta = \frac{2\sin^2\alpha}{2 - \sin^2\alpha} = \frac{2\sin^2 20°}{2 - \sin^2 20°} = 0.124$$

$$\therefore \quad 2\theta = 82.9° \quad \text{or} \quad \theta = 41.45°$$

and

$$\frac{d\omega_1}{dt} = \frac{\omega^2 \cos\alpha.\sin 2\theta.\sin^2\alpha}{(1 - \cos^2\theta.\sin^2\alpha)^2}$$

$$= \frac{(157)^2 \cos 20° \times \sin 82.9° \times \sin^2 20°}{(1 - \cos^2 41.45° \times \sin^2 20°)^2} = 3090 \text{ rad/s}^2 \quad \textbf{Ans.}$$

Maximum torque required

We know that maximum torque required

$$= I \times d\omega_1 / dt = 0.12 \times 3090 = 371 \text{ N-m} \quad \textbf{Ans.}$$

2) The angle between the axes of two shafts connected by Hooke's joint is 18°. Determine the angle turned through by the driving shaft when the velocity ratio is maximum and unity.

Solution. Given : $\alpha = 98°$

Let θ = Angle turned through by the driving shaft.

When the velocity ratio is maximum

We know that velocity ratio,

$$\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1 - \cos^2\theta . \sin^2\alpha}$$

The velocity ratio will be maximum when $\cos^2\theta$ is minimum, *i.e.* when

$$\cos^2\theta = 1 \quad \text{or} \quad \text{when } \theta = 0° \quad \text{or} \quad 180° \ \textbf{Ans.}$$

When the velocity ratio is unity

The velocity ratio (ω / ω_1) will be unity, when

$$1 - \cos^2\theta . \sin^2\alpha = \cos\alpha \qquad \text{or} \qquad \cos^2\theta = \frac{1 - \cos\alpha}{\sin^2\alpha}$$

$$\therefore \qquad \cos\theta = \pm\sqrt{\frac{1 - \cos\alpha}{\sin^2\alpha}} = \pm\sqrt{\frac{1 - \cos\alpha}{1 - \cos^2\alpha}} = \pm\sqrt{\frac{1}{1 + \cos\alpha}}$$

$$= \pm\sqrt{\frac{1}{1 + \cos 18°}} = \pm\sqrt{\frac{1}{1 + 0.9510}} = \pm 0.7159$$

$$\therefore \qquad \theta = 44.3° \quad \text{or} \quad 135.7° \ \textbf{Ans.}$$

3) Two shafts are connected by a Hooke's joint. The driving shaft revolves uniformly at 500 r.p.m. If the total permissible variation in speed of the driven shaft is not to exceed ±6% of the mean speed, find the greatest permissible angle between the centre lines of the shafts.

Solution. Given : $N = 500$ r.p.m. or $\omega = 2\,\pi \times 500 / 60 = 52.4$ rad/s

Let α = Greatest permissible angle between the centre lines of the shafts.

Since the variation in speed of the driven shaft is ± 6% of the mean speed (*i.e.* speed of the driving speed), therefore total fluctuation of speed of the driven shaft,

$$q = 12\ \%\ \text{of mean speed}\ (\omega) = 0.12\,\omega$$

We know that maximum or total fluctuation of speed of the driven shaft (q),

$$0.12\,\omega = \omega\left(\frac{1 - \cos^2\alpha}{\cos\alpha}\right) \quad \text{or} \quad \cos^2\alpha + 0.12\cos\alpha - 1 = 0$$

and

$$\cos\alpha = \frac{-0.12 \pm\sqrt{(0.12)^2 + 4}}{2} = \frac{-0.12 \pm 2.0036}{2} = 0.9418$$

...(Taking + sign)

$$\alpha = 19.64° \ \textbf{Ans.}$$

4) Two shafts are connected by a universal joint. The driving shaft rotates at a uniform speed of 1200 r.p.m. Determine the greatest permissible angle between the shaft axes so

that the total fluctuation of speed does not exceed 100 r.p.m. Also calculate the maximum and minimum speeds of the driven shaft.

Solution. Given : $N = 1200$ r.p.m.; $q = 100$ r.p.m.

Greatest permissible angle between the shaft axes

Let α = Greatest permissible angle between the shaft axes.

We know that total fluctuation of speed (q),

$$100 = N\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right) = 1200\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$$

$$\therefore \quad \frac{1-\cos^2\alpha}{\cos\alpha} = \frac{100}{1200} = 0.083$$

$$\cos^2\alpha + 0.083\cos\alpha - 1 = 0$$

and
$$\cos\alpha = \frac{-0.083 \pm \sqrt{(0.083)^2 + 4}}{2} = 0.9593 \quad \text{...(Taking + sign)}$$

$$\therefore \quad \alpha = 16.4° \quad \textbf{Ans.}$$

Maximum and minimum speed of the driven shaft

We know that maximum speed of the driven shaft,

$$N_{1\,(max)} = N / \cos\alpha = 1200 / 0.9593 = 1251 \text{ r.p.m.} \quad \textbf{Ans.}$$

and minimum speed of the driven shaft,

$$N_{1\,(min)} = N \cos\alpha = 1200 \times 0.9593 = 1151 \text{ r.p.m.} \quad \textbf{Ans.}$$

5) The driving shaft of a Hooke's joint runs at a uniform speed of 240 r.p.m. and the angle α between the shafts is 20°. The driven shaft with attached masses has a mass of 55 kg at a radius of gyration of 150 mm.

a. If a steady torque of 200 N-m resists rotation of the driven shaft, find the torque required at the driving shaft, when $\theta = 45°$.

b. At what value of 'α'will the total fluctuation of speed of the driven shaft be limited to 24 r.p.m ?

Solution. Given : $N = 240$ r.p.m or $\omega = 2\pi \times 240/60 = 25.14$ rad/s ; $\alpha = 20°$; $m = 55$ kg ; $k = 150$ mm $= 0.15$ m ; $T_1 = 200$ N-m ; $\theta = 45°$; $q = 24$ r.p.m.

1. *Torque required at the driving shaft*

Let $T' =$ Torque required at the driving shaft.

We know that mass moment inertia of the driven shaft,

$$I = m.k^2 = 55\,(0.15)^2 = 1.24 \text{ kg-m}^2$$

and angular acceleration of the driven shaft,

$$\frac{d\omega_1}{dt} = \frac{-\omega^2 \cos\alpha.\sin 2\theta.\sin^2\alpha}{(1-\cos^2\theta\sin^2\alpha)^2} = \frac{-(25.14)^2 \cos 20° \times \sin 90° \times \sin^2 20°}{(1-\cos^2 45° \sin^2 20°)^2}$$

$$= -78.4 \text{ rad/s}^2$$

$\therefore$ Torque required to accelerate the driven shaft,

$$T_2 = I \times \frac{d\omega_1}{dt} = 1.24 \times -78.4 = -97.2 \text{ N}-\text{m}$$

and total torque required on the driven shaft,

$$T = T_1 + T_2 = 200 - 97.2 = 102.8 \text{ N}-\text{m}$$

Since the torques on the driving and driven shafts are inversely proportional to their angular speeds, therefore

$$T'.\omega = T.\omega_1$$

or
$$T' = \frac{T.\omega_1}{\omega} = \frac{T\cos\alpha}{1-\cos^2\theta.\sin^2\alpha} \qquad \ldots\left(\because \frac{\omega_1}{\omega} = \frac{\cos\alpha}{1-\cos^2\theta.\sin\alpha}\right)$$

$$= \frac{102.8 \cos 20°}{1-\cos^2 45° \sin^2 20°} = 102.6 \text{ N-m} \quad \textbf{Ans.}$$

2. *Value of α for the total fluctuation of speed to be 24 r.p.m.*

We know that the total fluctuation of speed of the driven shaft (q),

$$24 = N\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right) = 240\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$$

or
$$\frac{1-\cos^2\alpha}{\cos\alpha} = \frac{24}{240} = 0.1$$

$$\cos^2\alpha + 0.1\cos\alpha - 1 = 0$$

$$\cos\alpha = \frac{-0.1 \pm \sqrt{(0.1)^2 + 4}}{2} = 0.95 \qquad \text{...(Taking + sign)}$$

$$\therefore \qquad \alpha = 18.2° \quad \textbf{Ans.}$$

Previous Problems

1) The driving shaft of a Hooke's Joint runs at uniform speed of 240 rpm and the angle between the shafts is 20^0. The driven shaft with attached masses has a mass of 55 kg at a radius of gyration of 150 mm. i) If a steady torque of 200 N-m resists rotation of the driven shaft, find the torque required at the driving shaft, when its angle of rotation $\Theta = 45^0$. ii) At what value of α will the total fluctuation of speed of the driven shaft be limited to 24 rpm. (2013-April-Set-1)

2) A Hooke's joint is used to connect two shafts whose axes are inclined at 20^0. The driving shaft rotates uniformly at 600 rpm. What are the extreme angular velocities of the driven shaft? Find the maximum value of retardation or acceleration and state the angle where both will occur. (2013-April-Set-2)

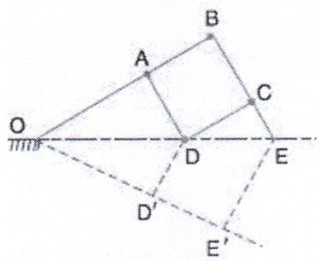
3) Two shafts are connected by a Hooke's Joint. The driving shaft revolves uniformly at 500 r.p.m. If the total permissible variation in speed of the driven shaft is not to exceed $\pm$ 6% of the mean speed, find the greatest permissible angle between the centre lines of the shafts. Also calculate the maximum and minimum speeds of the driven shaft. (2013-April-Set-3)

4) Two shafts are connected by a Hooke's Joint. The driving shaft revolves uniformly at 500 rpm. If the total permissible variation in speed of the driven shaft is not to exceed $\pm$6% of the mean speed, find the greatest permissible angle between the centre lines of the shafts (2014-Jan-Set-3)

Day-10

Q) Explain Pantograph?

A pantograph is an instrument used to reproduce to an enlarged or a reduced scale and as exactly as possible the path described by a given point.



It consists of a jointed parallelogram $ABCD$ as shown FIG. It is made up of bars connected by turning pairs. The bars BA and BC are extended to O and E respectively, such that

$$OA/OB = AD/BE$$

Thus, for all relative positions of the bars , the triangles OAD and OBE are similar and the points O, D and E are in one straight line. It may be proved that point E traces out the same path as described by point D.

From similar triangles OAD and OBE,

$$OD/OE = AD/BE$$

Let point O be fixed and the points D and E move to some new positions D' and E' . Then

$$OD'/OE' = OD/OE$$

This shows that the straight line DD' is parallel to the straight line EE' . Hence, if O is fixed to the frame of a machine by means of a turning pair and D is attached to a point in the machine which has rectilinear motion relative to the frame, then E will also trace out a straight line path. Similarly, if E is constrained to move in a straight line, then D will trace out a straight line parallel to the former.

Q) **What are different Straight Line Mechanisms and explain the Condition for Straight Line Motion Mechanisms Made up of Turning Pairs**?

One of the most common forms of the constraint mechanisms is that it permits only relative motion of an oscillatory nature along a straight line. The mechanisms used for this purpose are called straight line mechanisms . These mechanisms are of the following two types:

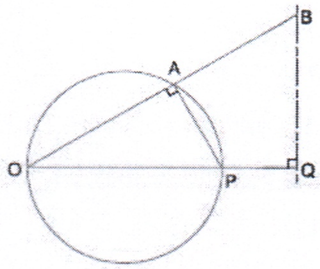
1. in which only turning pairs are used, and

2. in which one sliding pair is used.

These two types of mechanisms may produce exact straight line motion or approximate straight line motion.

Condition for Straight Line Motion Mechanisms Made up of Turning Pairs:

The principle adopted for a mathematically correct or exact straight line motion is described in Fig.



Let O be a point on the circumference of a circle of diameter OP. Let OA be any chord and B is a point on OA produced, such that $OA \times OB$ =constant

Then the locus of a point B will be a straight line perpendicular to the diameter OP.

Proof:

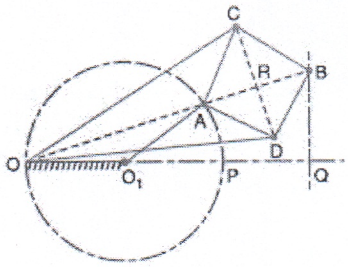
Draw BQ perpendicular to OP produced. Join AP. The triangles OAP and OBQ are similar.

OA/OQ= OP/OB => OA*OB=OP*OQ => OQ = OA*OB/OP

But OP is constant as it is the diameter of a circle, therefore, if $OA \times OB$ is constant, then OQ will be constant. Hence the point B moves along the straight path BQ which is perpendicular to OP

1. ***Peaucellier mechanism*** . It consists of a fixed link OO_1 and the other straight links O_1A , OC, OD, AD, DB, BC and CA are connected by turning pairs at their intersections, as shown in Fig. The pin at A is constrained to move along the circumference of a circle with the fixed diameter OP, by means of the link O_1A. In Fig.

$AC = CB = BD = DA$; $OC = OD$; and $OO_1 = O_1A$



It may be proved that the product $OA \times OB$ remains constant, when the link O_1A rotates. Join CD to bisect AB at R. Now from right angled triangles ORC and BRC, we have

$$OC^2 = OR^2 + RC^2 \quad ...(i)$$

$$BC^2 = RB^2 + RC^2 \quad ...(ii)$$

Subtracting equation **(*ii*)** from **(*i*),** we have

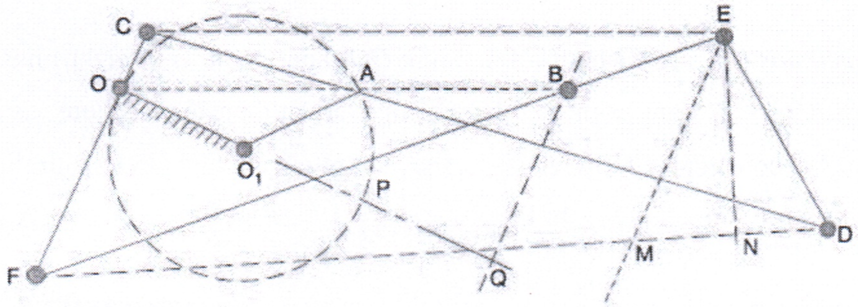
$$OC^2 - BC^2 = OR^2 - RB^2$$

$$= (OR + RB)\,(OR - RB)$$

$$= OB \times OA$$

Since OC and BC are of constant length, therefore the product $OB \times OA$ remains constant. Hence the point B traces a straight path perpendicular to the diameter OP.

2. ***Hart's mechanism*** . This mechanism requires only six links as compared with the eight links required by the Peaucellier mechanism. It consists of a fixed link OO_1 and other straight links O_1A , FC, CD, DE and EF are connected by turning pairs at their points of intersection, as shown in Fig. The links FC and DE are equal in length and the lengths of the links CD and EF are also equal. The points O, A and B divide the links FC, CD and EF in the same ratio. A little consideration will show that $BOCE$ is a trapezium and OA and OB are respectively parallel to FD and CE. Hence OAB is a straight line. It may be proved now that the product $OA \times OB$ is constant.



From similar triangles CFE and OFB,

$$\frac{CE}{FC} = \frac{OB}{OF} \quad \text{or} \quad OB = \frac{CE \times OF}{FC}$$

and from similar triangles FCD and OCA

$$\frac{FD}{FC} = \frac{OA}{OC} \quad \text{or} \quad OA = \frac{FD \times OC}{FC}$$

$$OA \times OB = \frac{FD \times OC}{FC} \times \frac{CE \times OF}{FC} = FD \times CE \times \frac{OC \times OF}{FC^2}$$

Since the lengths of OC, OF and FC are fixed, therefore

$$OA \times OB = FD \times CE \times \text{constant}$$

Now from point E, draw EM parallel to CF and EN perpendicular to FD. Therefore

$$FD \times CE = FD \times FM \quad \ldots(CE = FM)$$

$$= (FN + ND)(FN - MN) \quad \ldots(MN = ND)$$

$$= FN^2 - ND^2$$

$$= (FE^2 - NE^2) - (ED^2 - NE^2)$$

$$= FE^2 - ED^2 = \text{constant}$$

$$OA \times OB = \text{constant}$$

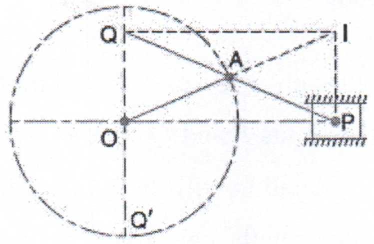
It therefore follows that if the mechanism is pivoted about O as a fixed point and the point A is constrained to move on a circle with centre O_1, then the point B will trace a straight line perpendicular to the diameter OP produced.

Day-11

Straight line motion mechanism with sliding pair i.e Scott Russell's Mechanism :

It consists of a fixed member and moving member P of a sliding pair as shown in Fig.

The straight link PAQ is connected by turning pairs to the link OA and the link P. The link OA rotates about O. A little consideration will show that the mechanism OAP is same as that of the reciprocating engine mechanism in which OA is the crank and PA is the connecting rod. In this mechanism, the straight line motion is not generated but it is merely copied.

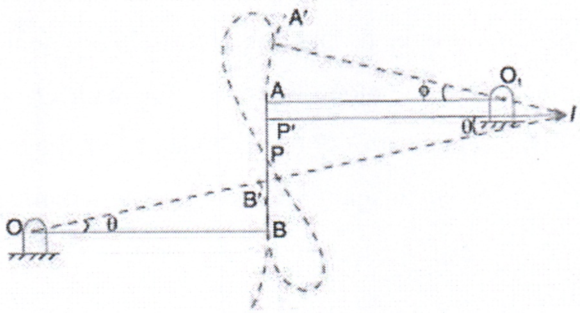


A is the middle point of *PQ* and *O A* = *AP* = *A Q*. The instantaneous centre for the link *PA Q* lies at *I* in *O A* produced and is such that *IP* is perpendicular to *OP*. Join *IQ*. Then *Q* moves along the perpendicular to *IQ*.

Since *OPIQ* is a rectangle and *IQ* is Perpendicular to *OQ*, therefore *Q* moves along the vertical line *OQ* for all positions of *QP*. Hence *Q* traces the straight line *OQ*

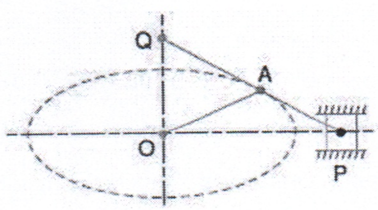
If *OA* makes one complete revolution, then *P* will oscillate along the line *OP* through a distance 2**OA* on each side of *O* and *Q* will oscillate along *OQ* through the same distance 2 **OA* above and below *O*. Thus, the locus of *Q* is a copy of the locus of *P*.

1. *Watt's mechanism* . It is a crossed four bar chain mechanism and was used by Watt for his early steam engines to guide the piston rod in a cylinder to have an approximate straight line motion.



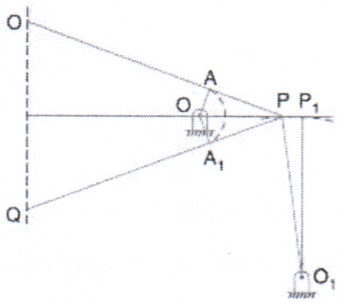
In Fig. $OBAO_1$ is a crossed four bar chain in which *O* and O_1 are fixed. In the mean position of the mechanism, links *OB* and O_1A are parallel and the coupling rod *A B* is perpendicular to O_1A and *OB*. The tracing point *P* traces out an approximate straight line over certain positions of its movement, if $PB/PA = O_1A/OB$.

2. *Modified Scott-Russel mechanism*.



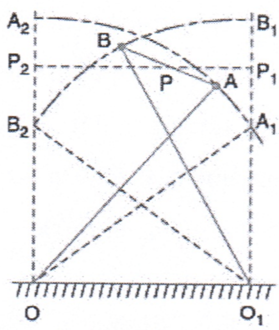
This mechanism, as shown in Fig. is similar to Scott-Russel mechanism discussed , but in this case AP is not equal to $A\ Q$ and the points P and Q are constrained to move in the horizontal and vertical directions. A little consideration will show that it forms an elliptical trammel, so that any point A on PQ traces an ellipse with semi -major axis $A\ Q$ and semi-minor axis AP.If the point A moves in a circle, then for point Q to move along an approximate straight line, the length OA must be equal $(A\ P)^2\ /(A\ Q)$. This is limited to only small displacement of P.

3. *Grasshopper mechanism* . This mechanism is a modification of modified Scott -Russel's mechanism with the difference that the point P does not slide along a straight-line, but moves in a circular arc with centre O.



It is a four bar mechanism and all the pairs are tur ning pairs as shown in Fig. In this mechanism, the centres O and O_1 are fixed. The link $O\ A$ oscillates about O through an angle AOA_1 which causes the pin P to move along a circular arc with O_1 as centre and O_1P as radius. For small angular displacements of OP on each side of the horizontal, the point Q on the extension of the link PA traces out an approximately a straight path QQ , if the lengths are such that $O\ A = (AP)^2\ /\ A\ Q$.

4. *Tchebicheff's mechanism* **.** It is a four bar mechanism in which the crossed links $O\ A$ and O_1B are of equal length, as shown in Fig.



The point P, which is the mid-point of $A\ B$ traces out an approximately straight line parallel to OO_1. The proportions of the links are, usually, such that point P is exactly above O or O_1 in the extreme positions of the mechanism *i.e.* when $B\ A$ lies along $O\ A$ or when $B\ A$ lies along BO_1.It may be noted that the point P will lie on a straight line parallel to OO_1, in the two extreme positions and in the mid position, if the lengths of the links are in proportions

$A B : OO_1 : O A = 1 : 2 : 2.5.$

Let AB= 1 Unit

$OA = O_1B = X$ Units ; $OO_1 = Y$ Units

When AB is extreme left position i.e A_2, B_2

In triangle $O\ O_1\ B_2$

$(O_1B_2)^2 - (OO_1)^2 = (O\ B_2)^2 = (OA_2 - A_2B_2)^2$

$X^2 - Y^2 = (X-1)^2$ Simplifying We get

$X = (Y^2 + 1)/2$ ...(i)

Let AB be in mean position which is parallel to OO_1 and from A draw a vertical line which intersects OO_1 at 'C'

From triangle OAC

$OA^2 - AC^2 = OC^2$

$OA^2 - OP_2^2 = AP_2^2$

$OA^2 - (OA_2 - A_2P_2)^2 = (PP_2 + AP)^2$

$x^2 - (x-0.5)^2 = (0.5y + 0.5)^2$

$x= (y^2/4)+0.5y+0.5$ ...(ii)

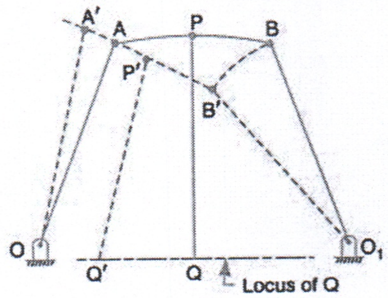
equating eqn (i) & (ii)

we get $Y = 2;\ X = 2.5$

Thus $A B : OO_1 : O A = 1 : 2 : 2.5.$

This ratio of the links ensures that P moves approximately in a horizontal straight line parallel to OO_1

5. *Roberts mechanism.* It is also a four bar chain mechanism, which, in its mean position, has the form of a trapezium. The links $O A$ and O_1B are of equal length and OO_1 is fixed. A barPQ is rigidly attached to the link $A B$ at its middle point P.



The mechanism is displaced as shown by the dotted lines in Fig. the point Q will trace out an approximately straight line.

Previous questions

1) A circle with EQ’ as diameter has a point Q on its circumference. P is a point on EQ’ produced such that if Q turns about E, the product of EQ x EP is constant. Prove that the point P moves in a straight line perpendicular to EQ’. (2013-April-Set-1)

2) Sketch a Pantograph and explain how the mechanism would be used to enlarge a drawing. (2013-April-Set-1)

3) Prove that a point on one of links of a Hart mechanism traces a straight line on the movement of its links? (2013-April-Set-2)

4) What is a pantograph and what are its uses? Explain the working of pantograph Mechanism. (2013-April-Set-2)

5) What are the applications of Pantograph? (2013-April-Set-3)

6) Derive the condition of exact straight line motion. Describe a mechanism consisting of turning pairs only giving a straight line motion to a point. (2013-April-Set-3)

7) Sketch and explain the following mechanisms: (2013-April-Set-4)

i) Pantograph ii) Robert’s mechanism iii) Grasshopper mechanism