Boundary layer theory

→ ~~L. prandtl~~ L. prandtal is defined the boundary layer in (1904)

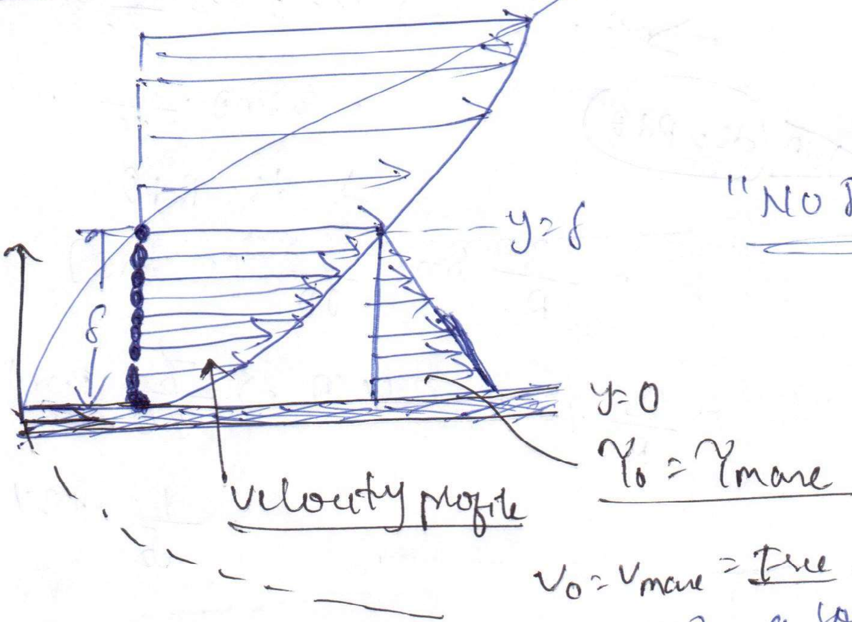
Before 1904

predicted value

v

Achieved value

= D'Alembert's principal



"No slip"

* whenever real fluid flows over a solid boundary because of "No slip" condition fluid particle will get slip to the boundary hence the velocity particle will be equal to velocity of the boundary

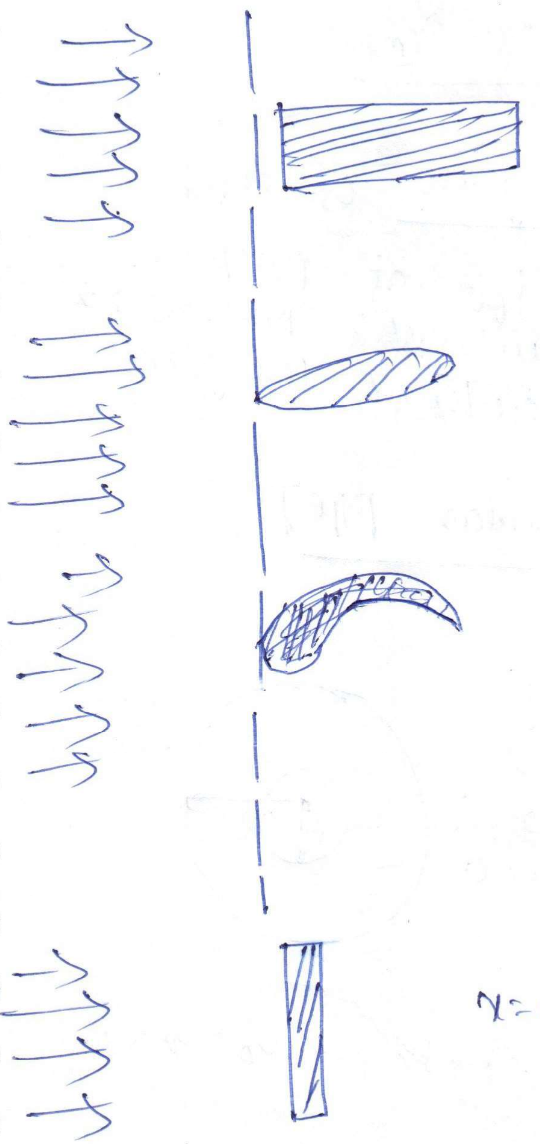
→ If the object is at rest fluid particle velocity near the boundary will be equal to velocity of the boundary (zero) and at a greater distance in normal direction particle velocity keeps on increasing and reaches maximum value and at distance of (δ) known as boundary layer thickness. This zone where velocity gradient exist is the boundary layer zone.

Boundary ~~layer~~ conditions :-

1) External flow

1. $y = 0$ Boundary, $V = 0$
2. $y = \delta$, $V = V_{max} = V_0$
3. $y = 0$, $\tau = \tau_0 = \tau_{max}$
4. $y = \delta$, $\tau = \tau_{min} = 0$
5. $x = 0$, $\delta = 0$

[∴ $\frac{du}{dy}$ = max^n] → (3) point



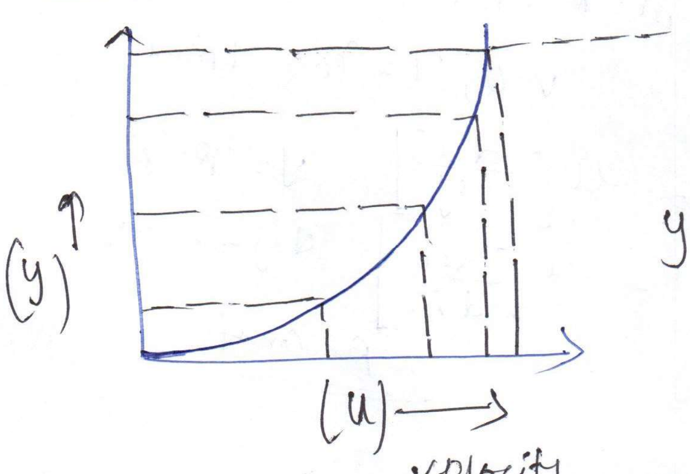
$x = 0$, $\delta = 0$

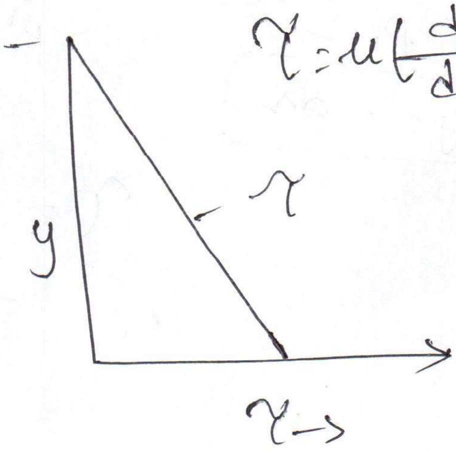
$x = 0$, $\delta = 0$ [streamline body]

$x = 0$, $\delta = 0$ (Aerofoil)

$x = 0$ $\delta \neq 0$ Blunt | rough | Blough body.

$\tau = \tau_{max}$:-





$\tau = \mu \left(\frac{du}{dy} \right)$

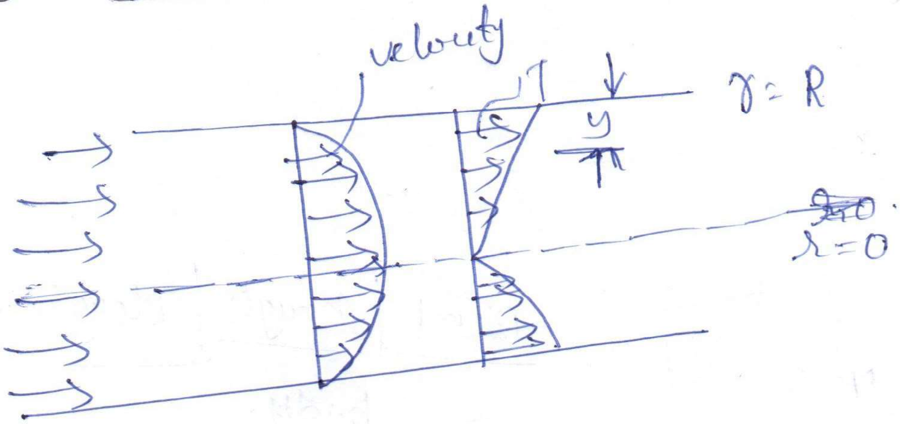
$\tau = \mu \left(\frac{du}{dy}\right)$ as $\frac{du}{dy} \to 0$ velocity gradient increase to max

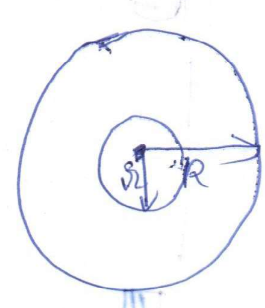
Hence $\tau = \tau_{max}$

100°C
} 5 sec
90°C
10% } 9 sec
80°C
|
|
|
40° } 10° } 2 Hr
30°C

→ Toughened glass is used in different purposes mostly automobile mirrors are manufactured the glass

(2) Internal flow [Through circular pipe]:-





$\boxed{\text{At } \tau = -\mu \cdot \frac{du}{dr}}$

→ $r = 0$, centre, $V = V_0 = V_{max}$

$\tau = \tau_{min} = 0$

$y = R - r$

$\boxed{dy = -dr}$

$r = R$, At Boundary

$V = 0$, $\tau = \tau_0 = \tau_{max}$

$\tau = \mu \left[\frac{du}{dy}\right]$ $y = R - r$

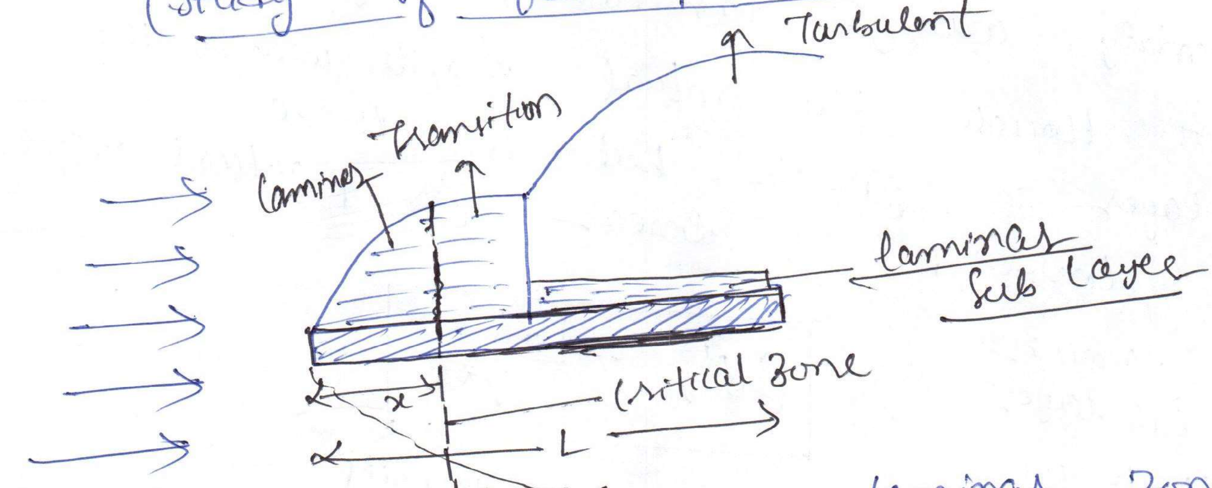
$dy = -dr$

$\tau = -\mu \left[\frac{du}{dr}\right]$

$R = \text{const}$

* Zones within boundary layer zone :-

(Study of flat plate kept at zero incidence)



1) **Laminar Zone :-**

→ velocity profile within the laminar zone is parabolic and is given by

$$\boxed{\frac{v}{V_0} = \left[\frac{y}{\delta}\right]^{n}}$$

where $n = 1$ to 3

If 'n' not given $\boxed{\frac{v}{V_0} = \frac{y}{\delta}}$

2) **Turbulent zone :-**

The velocity profile will be ~~Longitudinal~~ logarithmic and given by

$$\boxed{\frac{v}{V_0} = \left[\frac{y}{\delta}\right]^{n}} \qquad n = \frac{1}{7}$$

$$\therefore \frac{v}{V_0} = \left[\frac{y}{\delta}\right]^{1/7}$$

Velocity profile is governed by $\frac{1}{7}$th power low

(8) laminar sublayer zone [δ'] :-

within the turbulent zone near the boundary always laminar sublayer will exist though the actual profile with in the sublayer is parabolic but it will be considered as linear for practical calculation

laminar sub layer thickness => $\boxed{(\delta') = \frac{11.6\,\nu}{V^*}}$

$\nu = \mu/\rho$

ν = kinematic viscosity

V^* = shear velocity (velocity with which the shear plane is shifting)

$= \sqrt{\tau_0/\rho}$

$$\boxed{\delta' \propto \frac{1}{Re}}$$

* Limiting condition to be in laminar zone :-

(length of laminar zone)

I. The limiting condition given by critical eqn

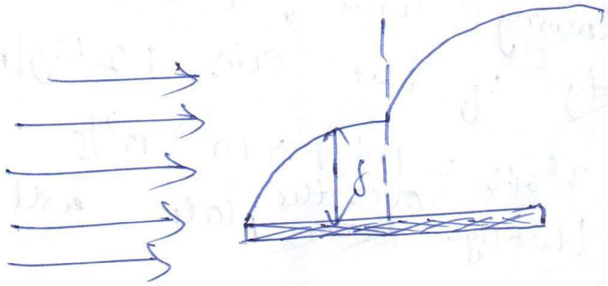
$(Re_{local})_{max} < Re_{critical}$

$Re_x < Re_{critical}$

$Re_{critical} < 5 \times 10^5$

$$\frac{\rho V x}{\mu} < 5 \times 10^5$$

* Boundary layer thickness (δ) :-



99% V_{max}

$y = \delta$,

$\tau = \tau_{min} = 0$

$$\boxed{\delta = \frac{5x}{\sqrt{\frac{\rho V x}{\mu}}}}$$

→ It is ~~the~~ a distance from the boundary to a point in normal direction where velocity reaches 99% of its maximum value

$$\delta_x = \frac{5x}{\sqrt{Re_x}} \longrightarrow \text{laminar} \text{ ---- } \delta_x \propto \sqrt{x}$$

$$\delta_x = \frac{0.376x}{(Re_x)^{1/5}} \longrightarrow \text{turbulent} \text{ ---- } \delta_x \propto x^{4/5}$$

$$\boxed{V_* = \sqrt{\left(\frac{\tau_0}{\rho}\right)} = \sqrt{\frac{F}{8}} \cdot V}$$

Note :- The above equations are based on Blausius experimentally result which can be used in absence of actual velocity profile

- The actual profile is known "von Karman" momentum integral equation to be used

$$\boxed{\frac{\tau_0}{\rho V_0^2} = \frac{d\theta}{dx}}$$

θ = Momentum thickness

→ $\delta = f[x, Re]$

→ $\delta = f[x, Re]$ ✓

$$\boxed{\theta = \int_0^\delta \frac{v}{V_0}\left[1 - \frac{v}{V_0}\right] dy}$$

IAS-07

A flat plate of 1 m wide & 2 m long is the boundary for air flowing with free stream velocity 6 m/sec & density of the air 1.2 kg/m3 & kinematic viscosity $\nu_{air} = 1.47 \times 10^{-5}\ m^2/s$ up to what length through over the plate will formly reet laminar.

ⓐ 1.208 m ⓑ 1.325 m

ⓑ 1.225 m ⓓ 1.475 m

Given data:-

1 m wide ✓
2 m long ✓

$\rho_{air} = 1.2\ kg/m^3$, $\nu_{air} =$ 1.47×10^{-5} m

Sol.

Flate plate

$x = ?$ $Re_x < 5 \times 10^5$

$$\frac{\rho V x}{\mu} < 5 \times 10^5$$

$$\frac{V x}{\nu} < 5 \times 10^5$$

$$x < \frac{5 \times 10^5 \times 1.47 \times 10^{-6}}{6}$$

$$x < 1.225$$

Q-03 civil

The laminar boundary layer thickness for a over a flat plate at pnt A is 2 cm & at a point B 1 m down stream of A is 3 cm then the distance of from leading edge of the plate

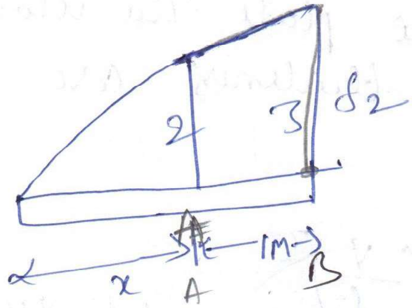
① 0.25 m ② 0.5 m ③ 0.8 m ④ 1.2 m

Sol.

A → 2 cm

B 1 m down stream of A is 3 cm

dist A = ?



$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{x_1}{x_2}}$$ Squaring

$$\left(2/3\right)^2 = \left(\sqrt{\frac{x}{x+1}}\right)^2$$

	A	B
$\delta \rightarrow$	2 CM	3 CM
dist	x	$x+1$

$$\frac{4}{9} = \frac{x}{x+1}$$

$$\delta \propto \sqrt{x}$$

$$x = 4/5 = 0.8$$

Q-12 For a flow air over a flat plate at a Location where boundary Reynolds number of 1000 at a Location where boundary thickness is ~~estimated~~ 4 MM If the velocity alone increased by a factor 4 Then the boundary layer thickness at that thickness is 'MM'

$$\boxed{\delta \propto \frac{1}{\sqrt{V}}}$$

Sol

	1	2
δ	4 MM	δ_2 ?
vel	V	4V

$$\delta = \frac{5x}{\sqrt{\frac{\rho V x}{\mu}}}$$

$\delta = 4$ MM

4V. $\delta_2 = ?$

$$\frac{\delta_2}{\delta_1} = \sqrt{\frac{V_1}{V_2}}$$

$$\delta_2 = 4 \times \sqrt{\frac{V}{4V}}$$

$$\delta_2 = 2\ \text{MM}$$

Q-04

Q G-04 For a flow over flate plate the velocity profile & boundary layer thickness are described by

$$\frac{u}{u_0} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \rightarrow \mu = 1.47 \times 10^{-5} \frac{N/sec}{m^2}$$

$$\rightarrow \delta = \frac{4.64x}{\sqrt{Re_x}} \qquad \rho = 1.2\,kg/m^3$$

→ The free stream velocity = 2 m/sec

wall shear stress at distance of 1 m from upstream end ——

$$\boxed{\tau_{0_{x=1}} = \tau_{y=0,\, x=1}}$$

Sol.

$$\tau_{0_{x=1}} = \mu\left[\frac{du}{dy}\right]_{x=1m,\, y=0}$$

$$\tau_0 = \mu \cdot \frac{d}{dy}\left[u_0\left(\frac{3}{2}\cdot\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right]\right]_{y=0}$$

$$= \mu\left[u_0\left(\frac{3}{2\delta} - \frac{1}{2}\frac{3y^2}{\delta^3}\right)\right]_{y=0}$$

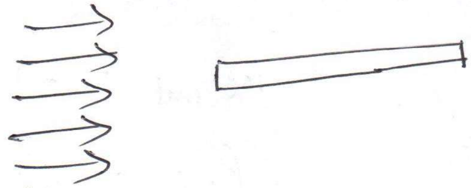
$$\Rightarrow \tau_0 = \frac{3\mu u_0}{2\delta}$$

$$\tau_0 = \frac{3\mu u_0}{\frac{2 \times 4.64 \times x}{\sqrt{Re_x}}} \Rightarrow \tau_0 = \frac{3\mu u_0}{\frac{2 \times 4.64 \times x}{\sqrt{\frac{\rho v x}{\mu}}}}$$

$$\tau_0 = \frac{3 \times 1.50 \times 10^{-5} \times 2}{\frac{2 \times 4.64 \times 1}{\sqrt{\frac{1.2 \times 2 \times 1}{1.50 \times 10^{-5}}}}} = 3.83 \times 10^{-3}\ N/m^2$$

$$= \frac{3 \times 1.50 \times 10^{-5} \times 2}{2 \times 4.64 \times 1 \sqrt{\frac{1.2 \times 2 \times 1}{1.5 \times 10^{-5}}}} = \underline{3.83 \times 10^{-3}\ N/m^2}$$

ESE



$$\frac{u}{u_0} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

V_0

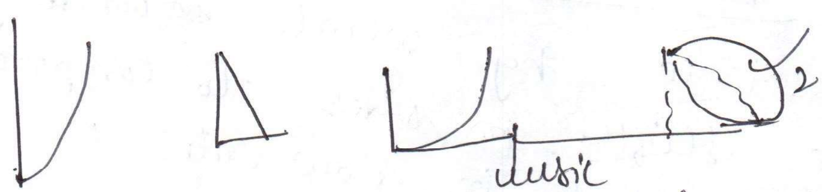
$$\frac{U}{U_0} = C_0 + C_1\left(\frac{y}{\delta}\right) + C_2\left(\frac{y}{\delta}\right)^2 + C_3\left(\frac{y}{\delta}\right)^3$$

→ Obtain → C_0 — (1) $y=0$, $u=0$ — (1)

→ Obtain τ prof commot — (2) $y=\delta$, $u=u_0=$

(3) $y=\delta$, $\tau=0$ — $\frac{du}{dy}=0$ — (3)

$\delta = f[x, Re]$ — (4) $y=0$ $\frac{d^2u}{dy^2}=0$ — (4)



cubic

$$\boxed{\frac{\tau_0}{\rho V_0^2} = \frac{d\theta}{dx}} \qquad \theta = \int_0^\delta \frac{V}{V_0}\left[1-\frac{V}{V_0}\right]dy$$

$$\frac{3\mu V_0}{2\delta} = \frac{d}{dx}\left[\frac{39\delta}{280}\right] \qquad \theta = \frac{39\delta}{280}$$

$$d\delta() = \int() dx$$

$$\delta = f(x) + C \rightarrow x=0,\ \delta=0 \rightarrow \delta_x = \frac{4.64x}{\sqrt{Re_x}}$$

DRDO-09

Q: Air flow past a golf ball of 20mm Radius, it is observed that that the flow becomes turbulent at reynolds no. of 2×10^5 $\rho = 1.2\ kg/m^3$ $\nu = 1.5\times10^{-5}\ m^2/sec$. find the velocity at which the flow becomes turbulent

Re crit = 2×10^5

$\rho_{air} = 1.2\ kg/m^3$

$\nu_{air} = 1.5\times10^{-5}\ m^2/sec$

$\nu_{air} = 1.5\times10^{-5}\ m^2/sec$

Sol → Re local < Re critical

→ $Re = \frac{\rho V D}{\mu} \leq 2\times10^5$

→ Re local < Re critical

$\frac{VD}{\nu} = \frac{\rho V D}{\mu} < 2\times10^5$

$V < \frac{2\times10^5 \times 1.5\times10^{-5}}{2\times20\times10^{-3}}$ $\Rightarrow V < 75\ m/s$

⇒ Displacement thickness (δ^*) :-

→ It is distance by which boundary has to be shifted in sides to ~~compensate~~ account compensate the lost in flow rate on of boundary layer formation

Lossin Q $\Rightarrow$ Displacement thickness (δ^*):-

$$\delta^* = \int_0^\delta \left[1 - \frac{v}{V_0}\right] dy$$

→ Momentum thickness :- θ,

→ Compensate loss in momentum

$$\theta = \int_0^\delta \frac{v}{V_0}\left[1 - \frac{v}{V_0}\right] dy$$

$Q = AV = V_{loss}$

Mom $= mv$

$KE = \frac{1}{2}mv^2$

Energy thickness :- (δ^E)

Compensate losing energy

$$\delta^E = \int_0^\delta \frac{v}{V_0}\left[1 - \left(\frac{v}{V_0}\right)^2\right] dy$$

$$\boxed{\delta^* > \delta^E > \theta}$$

$$\boxed{\text{Shape factor} = \frac{\delta^*}{\theta}}$$

Q $\Rightarrow$ $\boxed{\frac{v}{V_0} = \frac{y}{\delta}}$ → Flat plate at zero incidence under condn :-

$$\boxed{\delta^* , \delta^E , \theta}$$

Sol: → $\delta^* = \int_0^\delta \left[1 - \frac{v}{V_0}\right] dy = \int_0^\delta \left[1 - \frac{y}{\delta}\right] dy$

$$= \int_0^\delta \left[1 - \frac{y}{\delta}\right] dy$$

$$= \left[y - \frac{y^2}{2\delta}\right]_0^\delta = \left[\delta - \frac{\delta}{2}\right] = \delta/2$$

$$\theta = \int_0^\delta \frac{V}{V_0}\left[1 - \frac{V}{V_0}\right] dy$$

$$= \int_0^\delta \left[\frac{y}{\delta} - \frac{y^2}{\delta^2}\right] dy = \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right]_0^\delta$$

$$= \delta/6$$

$$\delta^E = \int_0^\delta \frac{V}{V_0}\left[1 - \left[\frac{V}{V_0}\right]^2\right] dy = \int_0^\delta \left[y/\delta - [y/\delta]^3 \right] dy$$

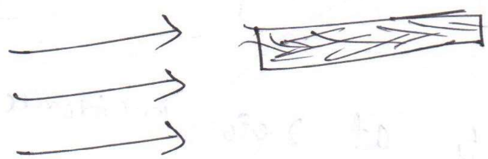
$$= \left[\frac{y^2}{2\delta} - \frac{y^4}{4\delta^3}\right]_0^\delta = \frac{\delta}{2} - \frac{\delta}{4} = \delta/4$$

$$\delta^* : \delta^E : \theta$$

$$\delta/2 : \frac{\delta}{4} : \frac{\delta}{6} \times 12$$

$$6 : 3 : 2$$

*



laminar flow over flat plate

① laminar, $\left(\frac{V}{V_0} = \frac{y}{\delta}\right)$

$$\delta^* = \delta/2, \ \delta^E = \frac{\delta}{4}; \ \theta = \delta/6$$

② Turbulent, $\frac{V}{V_0} = \left(\frac{y}{\delta}\right)^{1/7} = \left(\frac{y}{\delta}\right)^{1/m}$

$$\rightarrow \delta^* = \frac{\delta}{8} = \frac{\delta}{m+1}$$

③ $\tau = \tau_0\left[1 - \frac{y}{\delta}\right]$

$$\delta^* = \delta/3, \ \theta = \frac{2\delta}{15}$$

$\rightarrow \tau = \tau_0 [1 - y/\delta]$

$\rightarrow \mu\left(\frac{du}{dy}\right) = \tau_0 [1 - y/\delta]$

$\Rightarrow \int du = \frac{\tau_0}{\mu} \int (1 - y/\delta)\, dy$

$u = f(y) + \cancel{c}^{\,0}$ --- $y = 0,\ u = 0$

$u = f(y)$ — ①

$y = \delta;\ u = u_0$

$u_0 = (\ )$ — ②

① | ② $\boxed{\frac{u}{u_0} = \frac{2y}{\delta} - \frac{y^2}{\delta^2}}$

Q For a flow over flat plate velocity profile governing by 1/5th power law

Ratio of $\delta^*/\delta = ?$

Sol $\frac{\delta^*}{\delta} = \frac{1}{m+1} = \frac{1}{5+1}$

$\boxed{\frac{\delta^*}{\delta} = \frac{1}{6}}$ ✓

IAS
Q For flow over a flat plate at zero incidence with a free stream velocity of 10m/s at an $\rho_{air} = 1.2\ kg/m^3$ $\nu_{air} = 1.5 \times 10^{-5}\ m^2/sec$ was estimated instant displacement thickness as 0.5 mm where the boundary layer thickness was 2mm estimate the loss in flow rate kg/m. width on account of boundary layer formation per second.

a) 1.2×10^{-3} b) 5.0×10^{-3}
c) 6.0×10^{-3} d) 7.2×10^{-3}

Sol

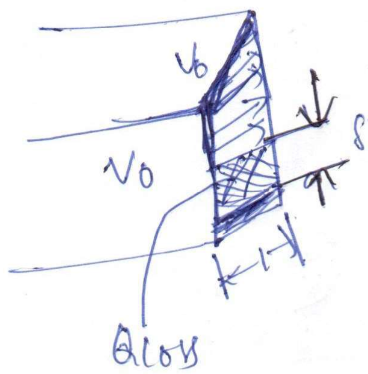
Q → $V_0 = 10\,m/s$ $\nu_{air} = 1.5 \times 10^{-5}\,m^2/sec$

in kg / per width

$\rho_{air} = 1.2\,kg/m^3$

$\delta = 2\,MM$

$\delta^* = 0.5\,MM$



→ $Q_{loss} = A_{loss} \times V_0$

$= [\delta^* \text{ width}] \times V_0$

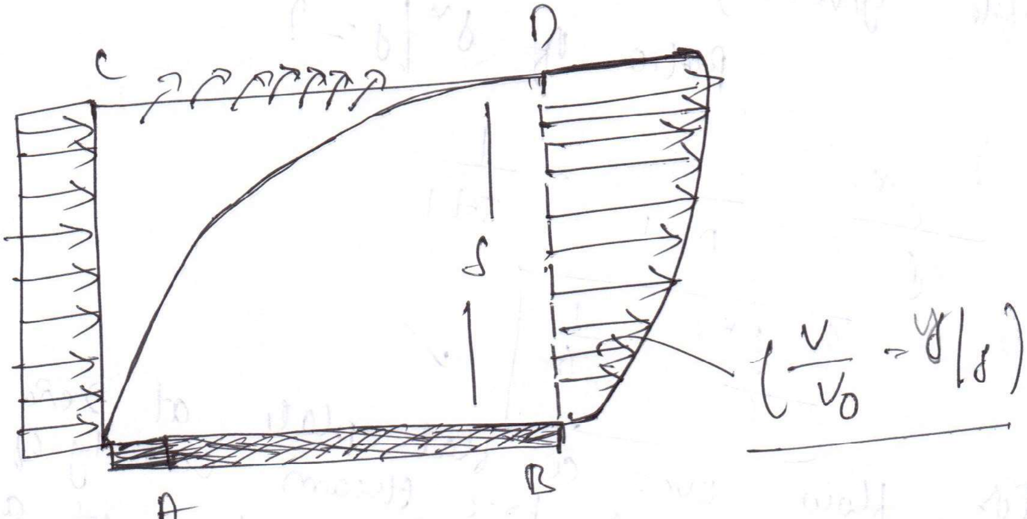
$Q_{loss} = 0.5 \times 10^{-3} \times 1 \times 10$

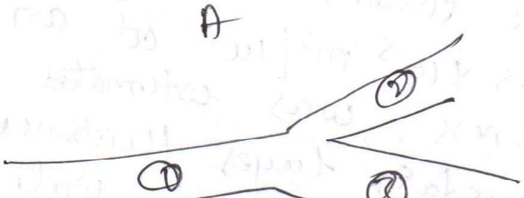
$= 5 \times 10^{-3}\,m^2/s$

→ $\rho \cdot Q_{loss} \cdot \dot{m}_{loss} = 1.2 \times 5 \times 10^{-3}\,kg/sec$

$= 6 \times 10^{-3}\,kg/sec$

Gate





$Q_1 = Q_2 + Q_3$

$Q_{CD} = ?$

$U_0 = 10\,m/sec$

$\delta = 2\,mm$

$Q_{AC} = Q_{CD} + Q_{BD}$

$Q_{AC} = Q_{CD} + Q_{BD}$

loss ← $Q_{CD} = Q_{AC} - Q_{BD}$

$Q_{CD} = Q_{AC} - Q_{BD}$ $\quad$ $Q_1 = Q_2 + Q_3$

loss

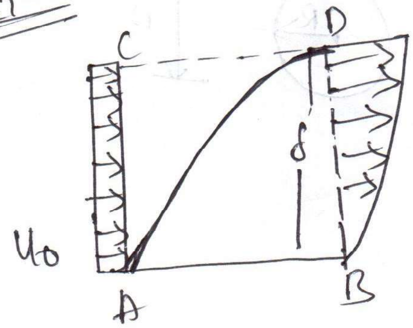
$$\boxed{\begin{aligned} Q_{loss} &= A_{loss} \times V_0 \\ &= (\delta^* \times 1) \times V_0 \end{aligned}}$$

$\Rightarrow \frac{\delta}{2} \times 1 \times V_0$

$= \frac{2}{2} \times 10^{-3} \times 1 \times 10$

$= 0.01\ m^3/sec$

Q-17



$$\frac{V}{V_0} = \frac{2y}{\delta} - \frac{y^2}{\delta^2}$$

$\delta^* = \delta/3$

$$\frac{Q_{AC}}{Q_{CD}} = \frac{\cancel{(\delta \times 1) \times U_0}}{\cancel{(\delta^* \times 1) \times U_0}}$$

loss

$\Rightarrow \frac{\delta}{\delta/3} = 3/2$