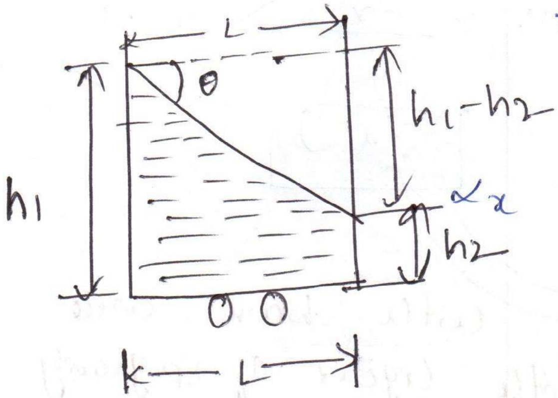
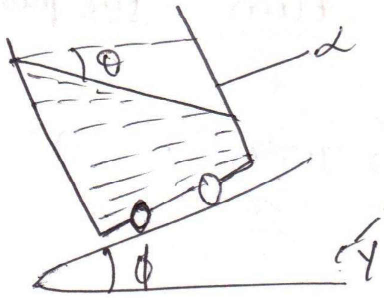
Liquids in Relative Motion (Moving fluids)

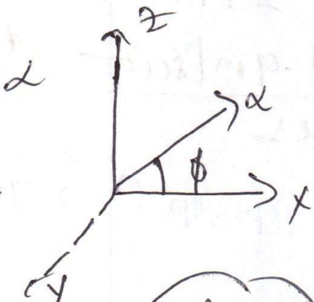
① Horizontal plane



→ $\boxed{\tan\theta = \left(\frac{h_1 - h_2}{L}\right) = \frac{\alpha_x}{g}}$

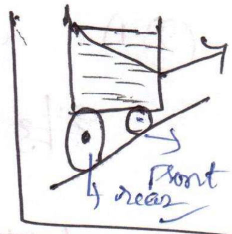
② Inclined plane :-



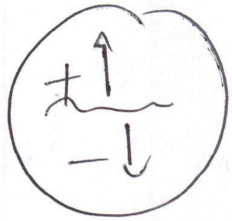


$\alpha_x = \alpha\cos\phi$

$\alpha_z = \alpha\cdot\sin\phi$



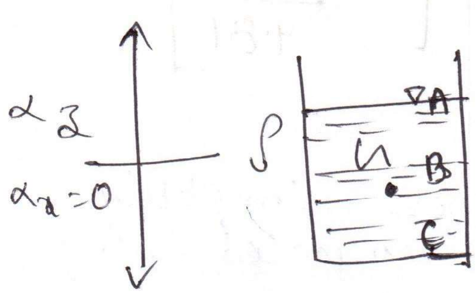
$\boxed{\tan\theta = \frac{\alpha_x}{g \pm \alpha_z}}$



③ Vertical plane :-

Motion of lift (or) Elevator

$P = \rho g h\left[1 + \frac{\alpha_z}{g}\right]$



$\boxed{P_A = P_B = P_C}$

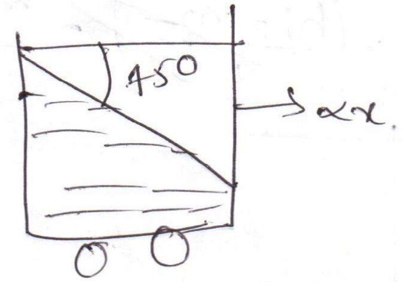
$\boxed{\alpha_z = g : (P_{atm})}$

Note:- If a container moving downwards with acceleration equal to ($\alpha_z = g$) Then the pressure at any point inside the liquid will be '0'

(1) The acceleration required to cause the free surface of liquid in a container moving on horizontal rails to dip by 45^0 is what is α_x = ?

(a) 1 (b) g (c) g/3 (d) 2g

Sol



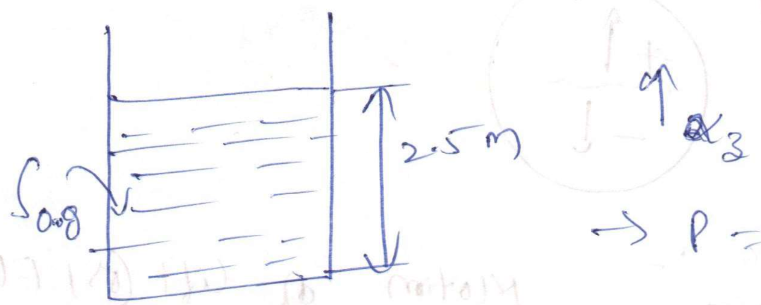
$$\tan\theta = \frac{\alpha_x}{g} = \tan 45^0 = 1$$

$$\boxed{\alpha_x = g}$$

(2) A Rectangular container with base area $2 \times 3 m^2$ was filled with liquid of sp. gravity 0.8 to a height 2.5 m and was imparted upward acceleration $4.9 m/sec^2$ then the pressure bottom of the container

(a) 9.5 kpa (b) 19.3 kpa (c) 29.4 kpa (d) 36.8 kpa

Sol:



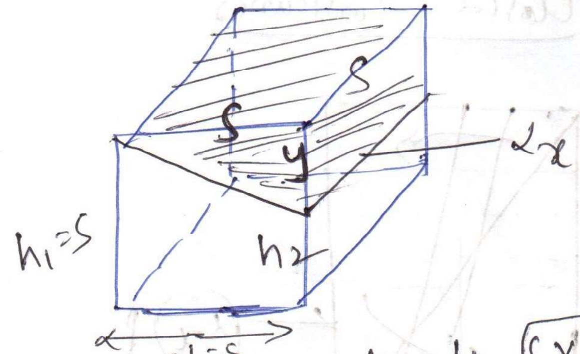
$$\rightarrow P = \rho g h\left[1 + \frac{\alpha_3}{g}\right]$$

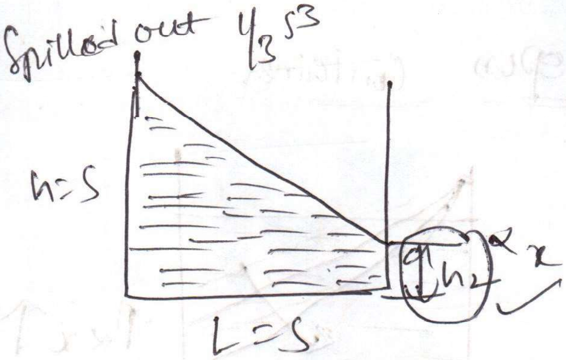
$$= 0.8 \times 1000 \times 9.81 \times 2.5\left[1 + \frac{4.9}{9.81}\right]$$

$$\simeq 29.4 \times 10^3 \; Pa$$

$$= 29.4 \; kpa$$

(8) An open cubical container completely filled with liquid was accelerated along one of its side on a horizontal plane such that find the acceleration imparted what $1/3$rd volume has been spilled out what is the acceleration required





Spilled out $= \frac{1}{2} \cdot (s \times y \times s) = \frac{1}{3} s^3 \Rightarrow y = 2s/3$

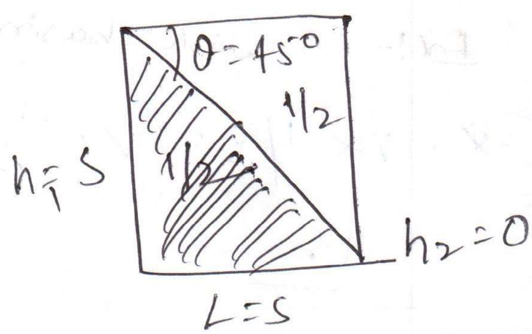
$h_1 = s \rightarrow h_2 = s - y = s/3$

$$\boxed{\tan\theta = \frac{\alpha_x}{g} = \frac{h_1 - h_2}{L}}$$

$\alpha_x = \left(\frac{s - h_2}{s}\right) g \Rightarrow \alpha_x = \left(\frac{s - s/3}{s}\right) g$

$$\boxed{\alpha_x = 2g/3}$$

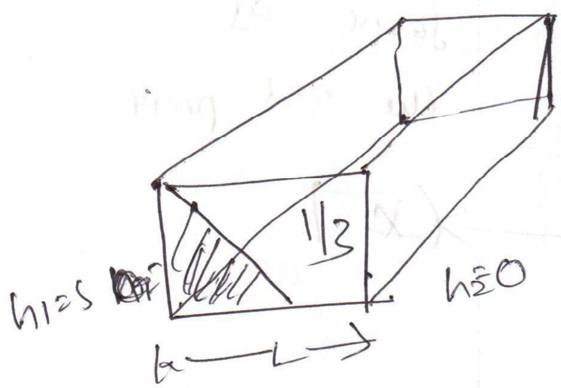
② $1/2$ vol spilled out



$\frac{\alpha_x}{g} = \frac{s - 0}{s} = 1$

$\alpha_x = g$

③ $2/3$ vol spilled out :-

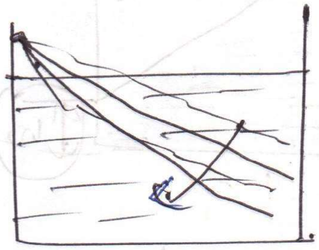


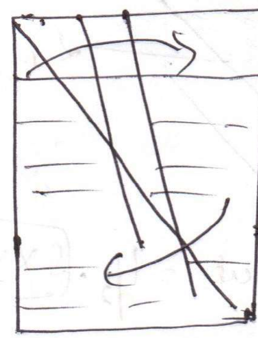
$\frac{\alpha_x}{g} = \frac{h_1 - h_2}{L} = \left(\frac{s}{L} - 0\right)$

$\alpha_x = \left(\frac{s}{2/3\, s} \times g\right)$

$\alpha_3 = 3g/2$

* Open Container — Closed Container





$F = m \cdot a_x$

$=$ Hyd. static pressure force

* Liquids in Rotation (vortex flow)

Forced vortex

* The Rotation is possible by providing externally energy

Ex : washing M/C

* $V \propto r$

$V = r\omega$

$= \pi DN/60$

* Flow of water in turbines/pumps

While in the impeller/Runner

After leaving the runner/Impeller → (X)

Free vortex

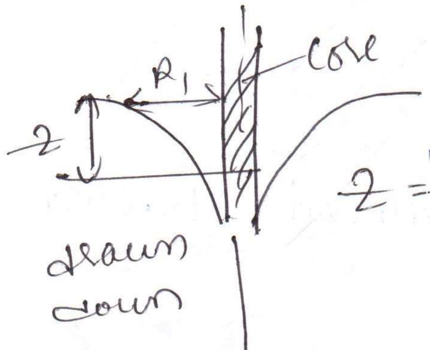
* Rotation is possible by conservation of angular momentum

Ex :- wash basin

* $V \propto 1/r \Rightarrow V = C/r$

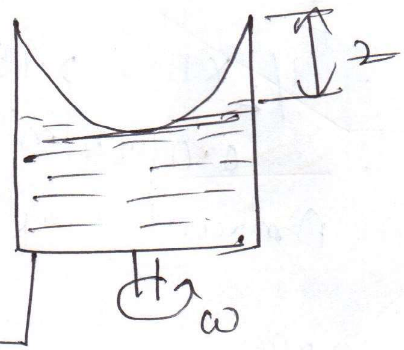
$V \cdot r = C$

* Same as the 3rd point



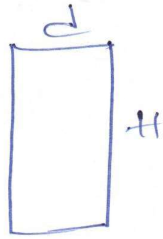
Forced vortex

$$z = \boxed{\frac{R^2\omega^2}{2g}}$$



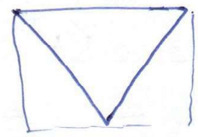
$$z = \frac{V^2}{2g}$$

→ Cylinder volume

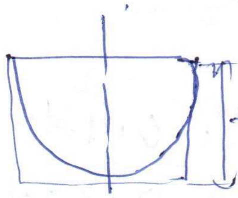


$$\rightarrow \boxed{\left(\frac{\pi d^2}{4}\right)\cdot H}$$

Cone:

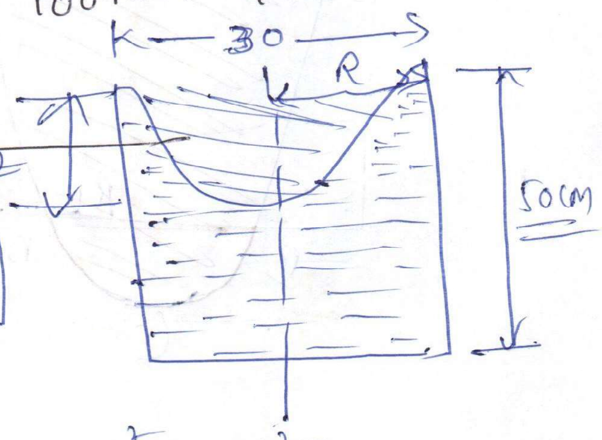


$$\frac{1}{3}\pi R^2 H$$



$$\boxed{\frac{1}{2}\pi R^2 H}$$

(1) An opened cylindrical container 30cm ~~radius~~ dia and 50cm height was completely filled with water & rotated about its axis. Find the amount of water spilled when the rotating at 180 RPM & 240 RM



Sol

$$\omega = \frac{2\pi N}{60}$$

$$= \frac{2\pi \times 180}{60} = 6\pi \text{ rad/sec}$$

vol spilled = vol paraboloid

$$= \boxed{\frac{1}{2}\pi R^2 z}$$

$$z = \frac{R^2\omega^2}{2g} \Rightarrow \frac{(0.15)^2(6\pi)^2}{2\times 9.81}$$

$$z = 0.407\,m$$

vol spilled $= \frac{1}{2} \pi R^2 z$

$= 1/2 \times \pi \times 0.15^2 \times 0.407$

$= 0.0144 m^3$

Amount of water spilled out $=$ 14.4 litre

$\to$ 2) $N = 240$ RPM

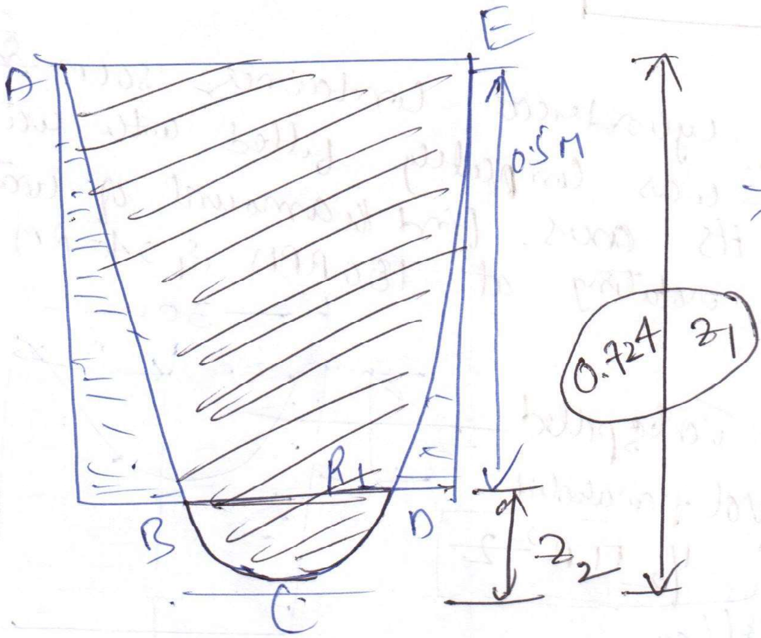
$\omega = \frac{2\pi N}{60} = \frac{2\pi \times \cancel{240}^{4}}{\cancel{60}} = 8\pi$ rad/sec

$z_1 = \frac{R^2\omega^2}{2g} = \frac{(0.15)^2(8\pi)^2}{2 \times 9.81} = 0.724 M$ ~~8π rad~~/sec

vol of spilled $= 1/2 \pi R^2 z_2$

$= 1/2 \pi \times 0.15^2 \phi 0.724$

$= 1/2 \times \pi \times 0.15^2 \times 0.724 = 0.0254 m^3$



$z_2 = 0.724 - 0.5$

$= 0.224$

$\to R_2^2 = \frac{2gz_2}{\omega^2}$

$R_2^2 = \frac{2 \times 9.81 \times 0.224}{\omega^2}$

$R_2^2 = \frac{2 \times 9.81 \times 0.224}{(8)^2}$

$= \frac{4.39488}{64}$

$= 0.06867$

$\boxed{2g z_2 / w^2}$

Vol spilled out

$= \frac{1}{2}\pi R_1^2 z_1 - \frac{1}{2}\pi R_2^2 z_2$

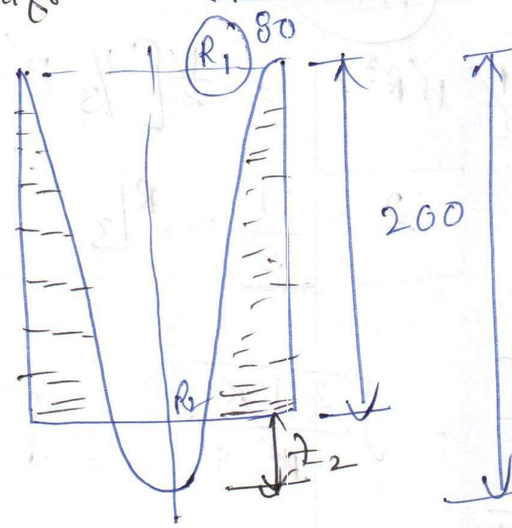
$= 0.0254 - \dfrac{\frac{1}{2}\pi \times 2 \times 9.81 \times (0.224)(0.06867)^2}{8\pi^2}$ $= \dfrac{6.903 \times (0.06867)^2}{8\pi^2}$

$= 0.0231 m^3$

$= 0.0231 m^3 \Rightarrow$ 23.1 litre

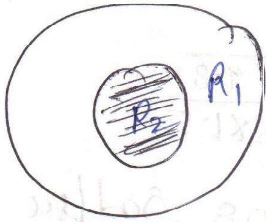
(2) A open cylindrical container with 80MM radius and 200MM height was rotated about its axis such that 1/3 rd of area at the base is exposed. Find the uniform speed of rotation?

Sol



$\frac{1}{3}\pi R_1^2 = \pi R_2^2$

$\boxed{R_2^2 = R_1^2/3}$



$z_1 = \dfrac{R_1^2 w_1^2}{2g}, \; z_2 = \dfrac{R_2^2 w^2}{2g}$

$z_1 - z_2 = \dfrac{R_1 w_1^2}{2g} - \dfrac{R_2^2 w_2^2}{2g} = 0.2$

$w = \sqrt{\dfrac{2g \times 0.2}{(R_1^2 - R_1^2/3)}}$

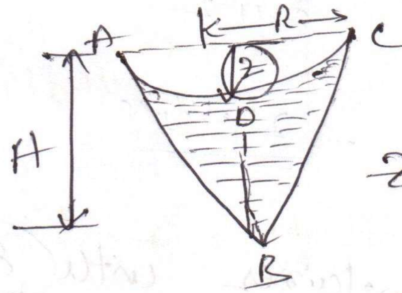
$w = \sqrt{\dfrac{2 \times 9.81 \times 0.2}{0.08^2 - \frac{0.08^2}{3}}}$

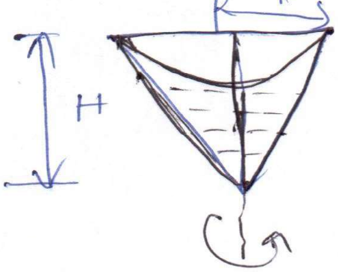
$w = 30.26$ rad/sec

IAS

A right circular conical container with apex downwards and axis vertical was exactly half full. Find the uniform speed rotation about its axis such that when water is about to spill?

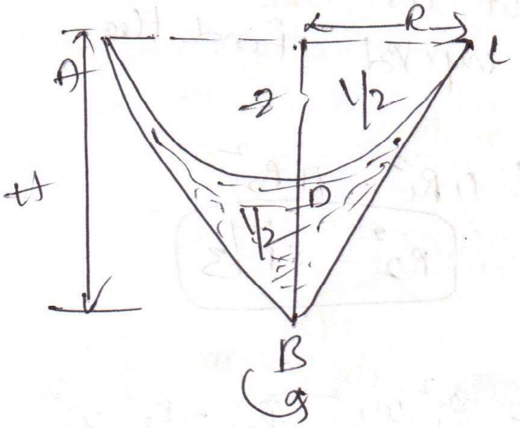
Sol:





$$Z = \frac{R^2\omega^2}{2g}$$

$$\omega = \sqrt{\frac{2gZ}{R^2}}$$



(R, Z) (R, H)

Vol. paraboloid (ADC) = 1/2 vol cone (ABC)

$$\cancel{\tfrac{1}{2}}\cancel{\pi}\cancel{R^2}Z = \cancel{\tfrac{1}{2}}\left[\tfrac{1}{3}\cancel{\pi}\cancel{R^2}H\right]$$

$$Z = \frac{H}{3} = R/3$$

$$\omega = \sqrt{\frac{2gZ}{R^2}} = \sqrt{\frac{2gR}{R^2 3}} = \sqrt{2g/3R}$$

$$\omega = \sqrt{2g/3}$$

$$\omega = \sqrt{2g/3R} \Rightarrow \sqrt{\frac{2\times 9.81}{3\times 1.5}}$$

$$\omega = 2.08 \text{ Rad/sec}$$